A Feasibility Study on the Position Hypothesis Based RTK with the Aids of 3D Building Models

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Precise GNSS Positioning, ION ITM 2021, 25-28 Jan 2021, Virtual

Centimetre-Level Positioning

New era!!

Land surveying

Unmanned aerial vehicle (UAV) delivery

Autonomous driving

Centimetre accuracy is required

Real-Time Kinematic (RTK) GNSS

Urban GNSS Positioning

Line-of-sight (LOS) pseudorange:

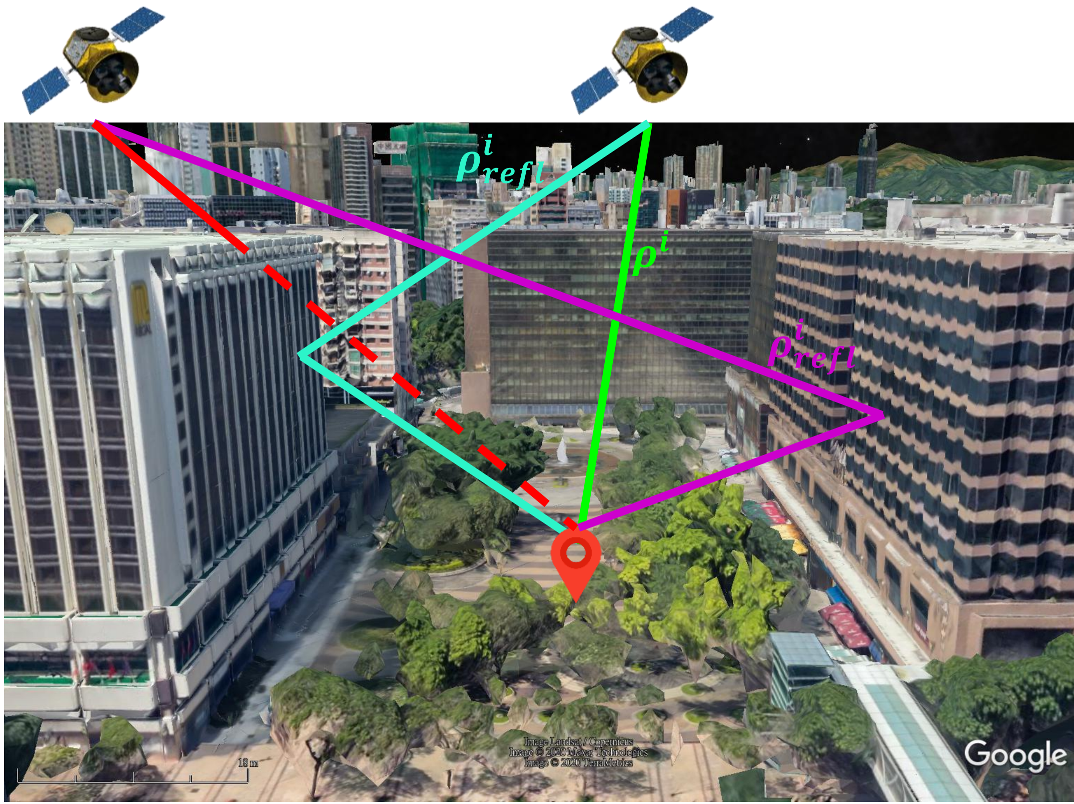
$$\rho^i = (t_{rx} - t_{tx})c$$

Reflected signal:

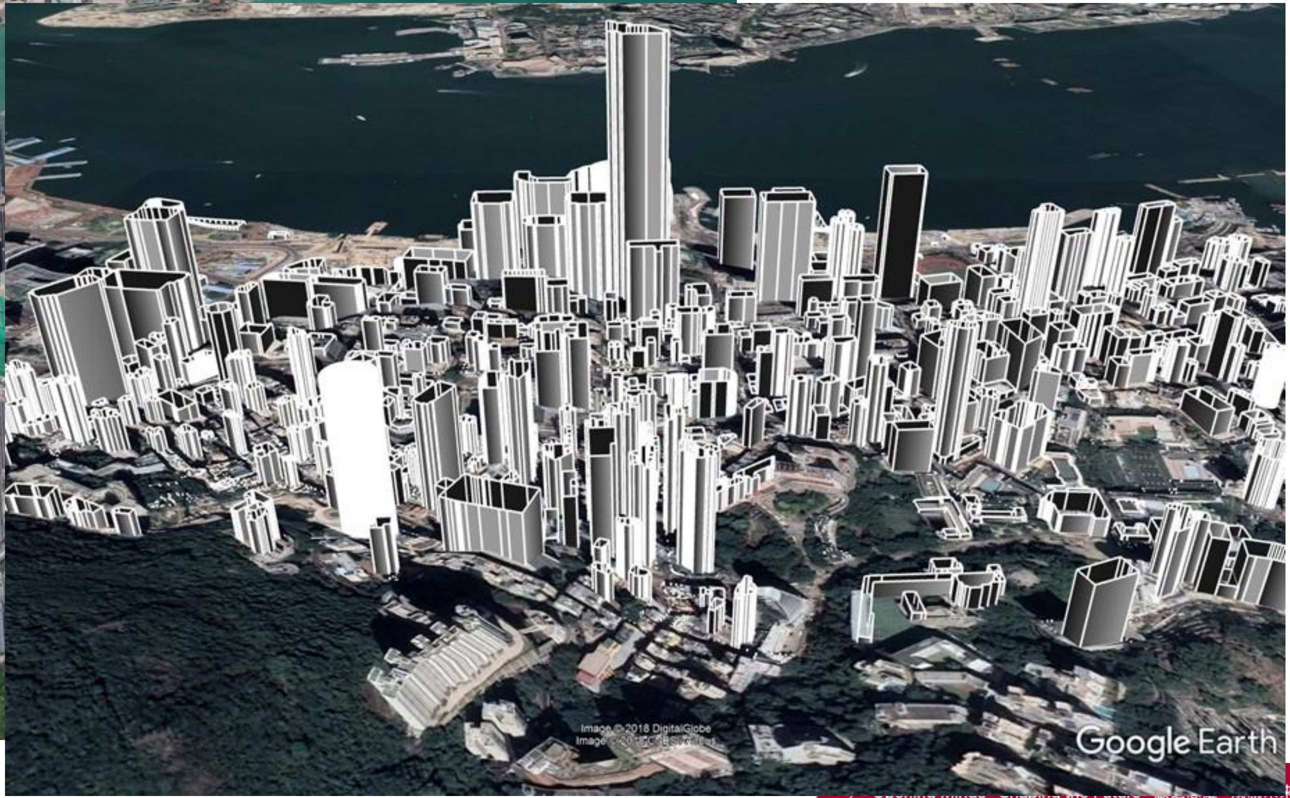
$$\rho^i_{refl} = (t_{rx} - t_{tx} + t_{refl})c$$

NLOS reception: LOS signal is blocked only receiving reflected signal

Multipath: receiving both LOS signal and reflected signal

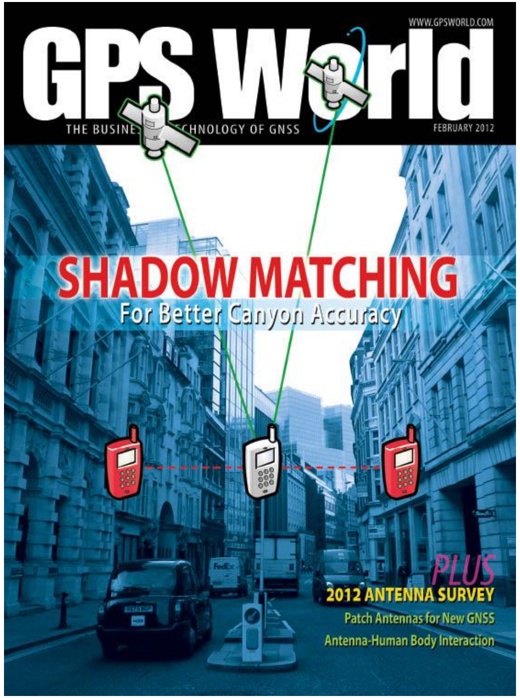


Widely available 3D building model now!



Popular 3D Mapping Aided (3DMA) GNSS

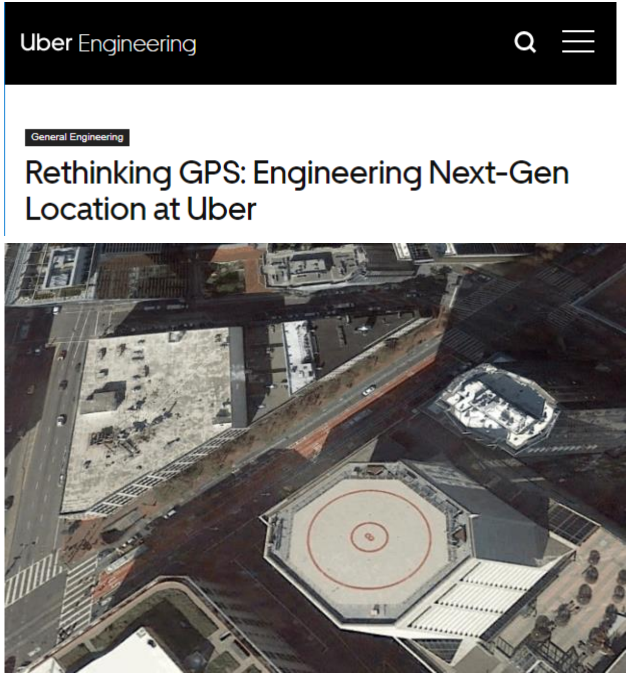
Shadow matching
(Satellite Visibility)



+

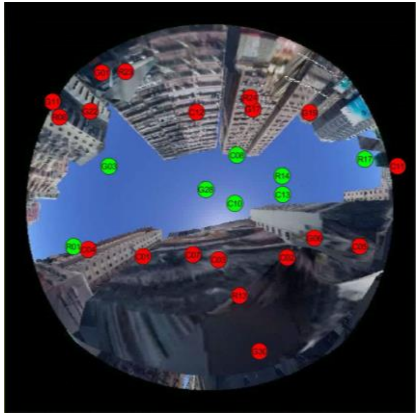
GNSS Ray-tracing
(Range and C/N_0)

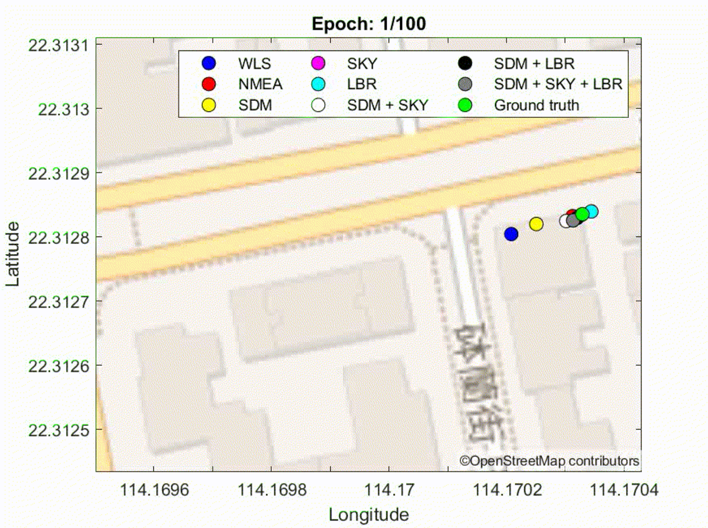
=



[Rethinking GPS: Engineering Next-Gen Location at Uber]

Ranging 3DMA GNSS Performance

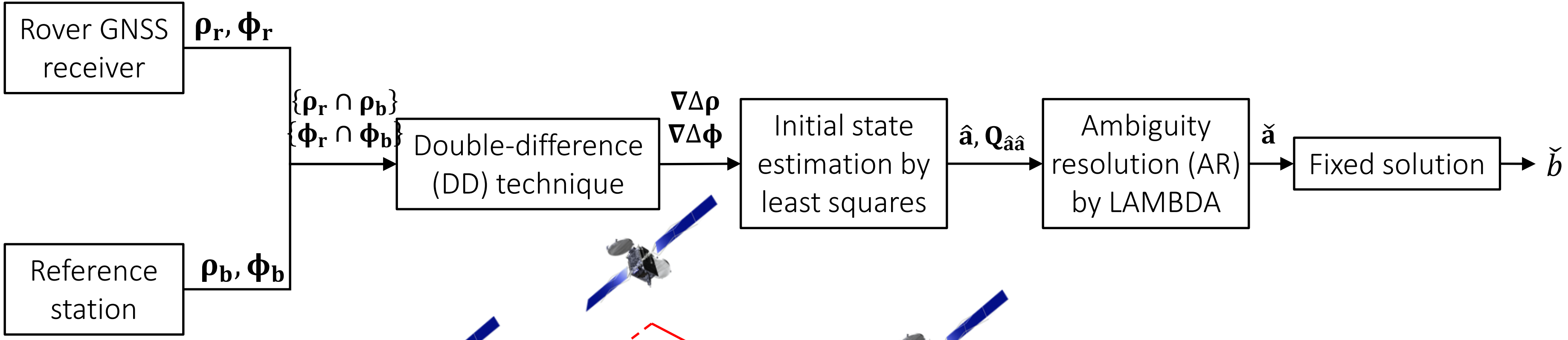


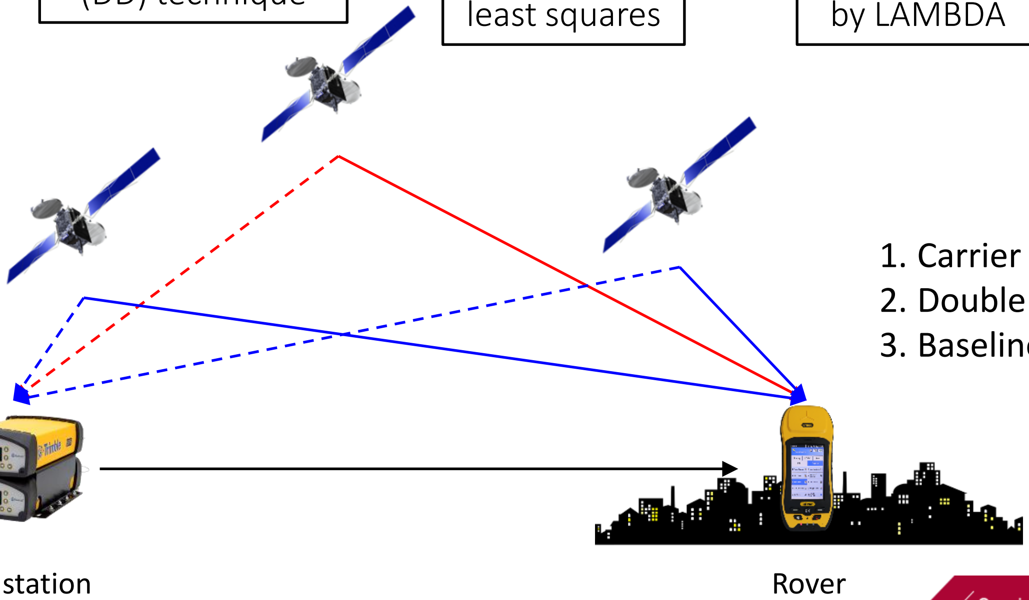


RMS error (m)	NMEA	WLS	SDM	LBR	SKY	SDM + LBR	SDM + SKY	SDM + LBR + SKY
2D	6.64	18.33	5.68	5.65	6.31	4.89	5.21	5.27
Along street	3.39	14.57	4.51	5.01	5.75	4.67	4.93	4.90
Across street	5.70	11.12	3.45	2.61	2.60	1.45	1.69	1.95

[4] H.-F. Ng, G. Zhang, L.-T. Hsu, "GNSS NLOS Pseudorange Correction based on Skymask for Smartphone Applications," *Proceedings of the 32nd International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2019)*, Miami, Florida, September 2019, pp. 109-119. doi: https://doi.org/10.33012/2019.17121

Conventional RTK GNSS with LAMBDA

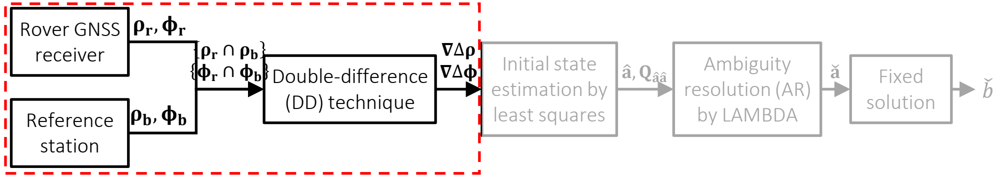




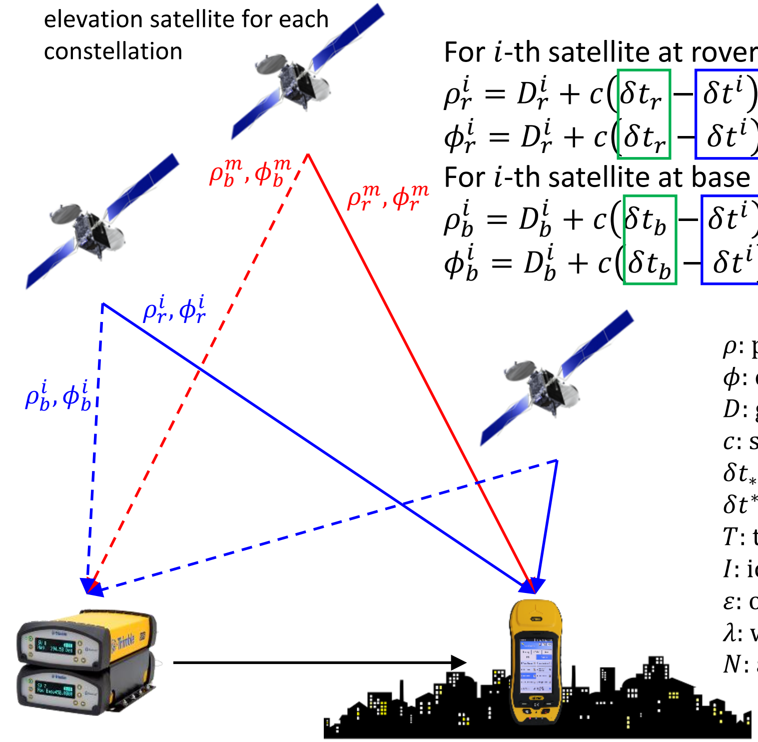
1. Carrier phase measurements
2. Double differencing
3. Baseline vector

[5] P. J. G. Teunissen, "Least-Squares Estimation of the Integer GPS Ambiguities," *Invited lecture, section IV theory and methodology, IAG general meeting*, 1993.

Double-Differencing (DD)



Master satellite: highest elevation satellite for each constellation



Base station

Rover

For i-th satellite at rover $*_r$

$$\rho_r^i = D_r^i + c(\delta t_r - \delta t^i) + T_r^i + I_r^i + \varepsilon_r^i$$
$$\phi_r^i = D_r^i + c(\delta t_r - \delta t^i) + T_r^i - I_r^i + \varepsilon_r^i + \lambda^i N_r^i$$

For i-th satellite at base station $*_b$

$$\rho_b^i = D_b^i + c(\delta t_b - \delta t^i) + T_b^i + I_b^i$$
$$\phi_b^i = D_b^i + c(\delta t_b - \delta t^i) + T_b^i - I_b^i + \lambda^i N_b^i$$

Single differencing between rover and base station

$$\Delta\rho^i = \rho_r^i - \rho_b^i = D_r^i - D_b^i + c(\delta t_r - \delta t_b) + \varepsilon_r^i$$
$$\Delta\phi^i = \phi_r^i - \phi_b^i = D_r^i - D_b^i + c(\delta t_r - \delta t_b) + \varepsilon_r^i + \lambda^i N_r^i$$

Similarly at master satellite

$$\Delta\rho^m = \rho_r^m - \rho_b^m = D_r^m - D_b^m + c(\delta t_r - \delta t_b) + \varepsilon_r^m$$
$$\Delta\phi^m = \phi_r^m - \phi_b^m = D_r^m - D_b^m + c(\delta t_r - \delta t_b) + \varepsilon_r^m + \lambda^m N_r^m$$

- ρ: pseudorange observation (m)
- ϕ: carrier phase observation (m)
- D: geometric distance (m)
- c: speed of light (m/s)
- δt_*: receiver clock delay (s)
- δt^*: satellite clock delay (s)
- T: tropospheric error (m)
- I: ionospheric error (m)
- ε: other error term (m)
- λ: wavelength (m)
- N: ambiguity (cyc)

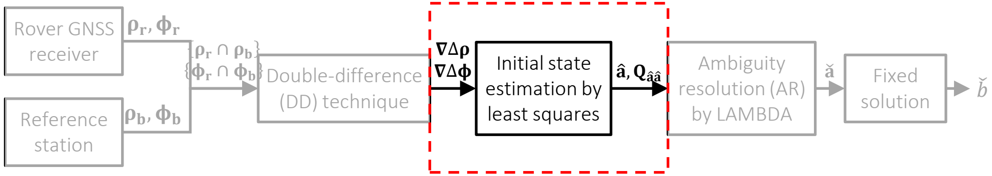
Double differencing between i -th and master satellite

$$\nabla\Delta\rho^i = \Delta\rho^i - \Delta\rho^m = D_r^i - D_b^i + \varepsilon_r^i$$
$$\nabla\Delta\phi^i = \Delta\phi^i - \Delta\phi^m = D_r^i - D_b^i + \varepsilon_r^i + \lambda^i N_r^i - \lambda^m N_r^m$$

Simplified,

$$\nabla\Delta\rho^i = \nabla\Delta D^i + \varepsilon_{\rho^i}$$
$$\nabla\Delta\phi^i = \nabla\Delta D^i + \lambda^i \nabla\Delta N^i + \varepsilon_{\phi^i}$$

Least Square Estimation



Measurement vector (**y**) = Design matrix (**A**) State vector (**x**)

$$\underbrace{\begin{bmatrix} \nabla\Delta\rho^1 - \nabla\Delta D^1 \\ \vdots \\ \nabla\Delta\rho^i - \nabla\Delta D^i \\ \nabla\Delta\phi^1 - \nabla\Delta D^1 \\ \vdots \\ \nabla\Delta\phi^i - \nabla\Delta D^i \end{bmatrix}}_{\mathbf{y}} = \underbrace{\begin{bmatrix} \mathbf{u}_\mathbf{r}^\mathbf{1} - \mathbf{u}_\mathbf{r}^\mathbf{m} & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{u}_\mathbf{r}^\mathbf{i} - \mathbf{u}_\mathbf{r}^\mathbf{m} & 0 & \dots & 0 \\ \mathbf{u}_\mathbf{r}^\mathbf{1} - \mathbf{u}_\mathbf{r}^\mathbf{m} & \lambda^1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{u}_\mathbf{r}^\mathbf{i} - \mathbf{u}_\mathbf{r}^\mathbf{m} & 0 & \dots & \lambda^i \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} b_x \\ b_y \\ b_z \\ \nabla\Delta N^1 \\ \vdots \\ \nabla\Delta N^i \end{bmatrix}}_{\mathbf{x}}$$

Baseline vector $\hat{\mathbf{b}}$ (b_x, b_y, b_z)

Float DD ambiguities $\hat{\mathbf{a}}$ ($\nabla\Delta N^1 \dots \nabla\Delta N^i$)

Where $\mathbf{u}_\mathbf{r}^* = \frac{\mathbf{p_r} - \mathbf{p}^*}{D_r^*} = \left[\frac{p_{r,x} - p_x^*}{D_r^*}, \frac{p_{r,y} - p_y^*}{D_r^*}, \frac{p_{r,z} - p_z^*}{D_r^*}\right]$

Float solution $\hat{\mathbf{x}} = \mathbf{N}^{-1}\mathbf{A}^T\mathbf{Q}^{-1}\boldsymbol{y} = \left(\mathbf{A}^T\mathbf{Q}^{-1}\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{Q}^{-1}\boldsymbol{y}$

Normalized weighted sum of the squared measurement residuals of LS

$$\hat{\delta}^2 = \frac{(\mathbf{y} - \hat{\mathbf{y}})^T\mathbf{Q}^{-1}(\mathbf{y} - \hat{\mathbf{y}})}{s - u}$$

Cholesky factorization

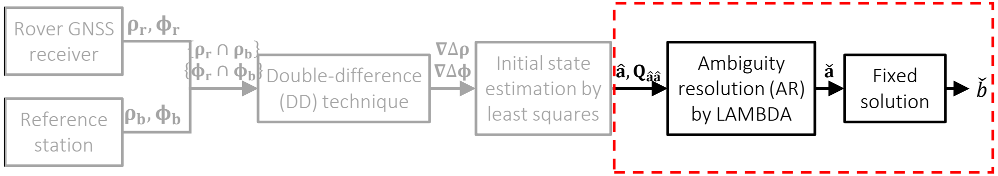
$$U = chol\left(\hat{\delta}^2\mathbf{N}^{-1}\right)$$

Variance-covariance (VC) matrix

$$\mathbf{C} = \mathbf{U}^T\mathbf{U} = \begin{bmatrix} \mathbf{Q}_{\hat{\mathbf{b}}\hat{\mathbf{b}}} & \mathbf{Q}_{\hat{\mathbf{b}}\hat{\mathbf{a}}} \\ \mathbf{Q}_{\hat{\mathbf{a}}\hat{\mathbf{b}}} & \mathbf{Q}_{\hat{\mathbf{a}}\hat{\mathbf{a}}} \end{bmatrix}$$

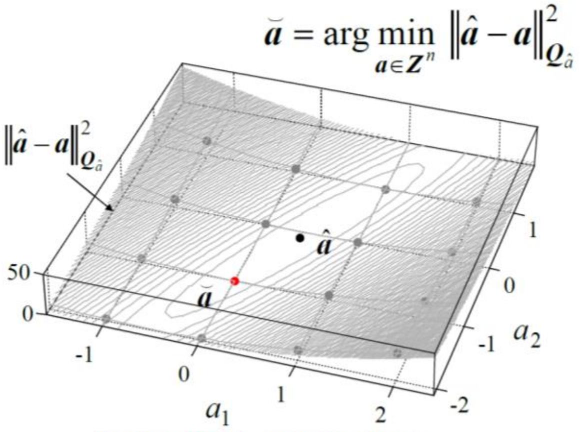
$$\hat{\boldsymbol{a}} \longrightarrow \check{\mathbf{a}} \in \mathbb{Z} \Rightarrow \check{\mathbf{p}} = \hat{\mathbf{p}} + \check{\boldsymbol{b}}$$

Ambiguity Resolution & Fixed Solution



Input $\hat{\mathbf{a}}, \mathbf{Q}_{\hat{\mathbf{a}}\hat{\mathbf{a}}}$ → **Objective function** $\min_{\mathbf{a}} \|\hat{\mathbf{a}} - \mathbf{a}\|^2_{\mathbf{Q}^{-1}_{\hat{\mathbf{a}}\hat{\mathbf{a}}}}$ where $\mathbf{a} \in Z^n$ → **Integer searching** $(\hat{\mathbf{a}} - \mathbf{a})^* \mathbf{Q}^{-1}_{\hat{\mathbf{a}}\hat{\mathbf{a}}} (\hat{\mathbf{a}} - \mathbf{a}) \leq \chi^2$ → **Fixed baseline solution** $\check{\mathbf{b}} = \hat{\mathbf{b}} - \mathbf{Q}_{\hat{\mathbf{b}}\hat{\mathbf{a}}} \mathbf{Q}^{-1}_{\hat{\mathbf{a}}\hat{\mathbf{a}}} (\hat{\mathbf{a}} - \check{\mathbf{a}})$

Note: $\mathbf{a}$ can be transformed into $\boldsymbol{z}$ via Z-transformation



Problem: Nsv ↑ , searching time ↑

Integer least-squares (ILS) [6]

Ratio test

$$\frac{R_2}{R_1} > \mathrm{k} \quad \text{where } \mathrm{k} = 3$$

with $R_1 = (\hat{\mathbf{a}} - \check{\mathbf{a}}_1)^{\mathrm{T}} \mathbf{Q}^{-1}_{\hat{\mathbf{a}}\hat{\mathbf{a}}} (\hat{\mathbf{a}} - \check{\mathbf{a}}_1)$ and

$R_2 = (\hat{\mathbf{a}} - \check{\mathbf{a}}_2)^{\mathrm{T}} \mathbf{Q}^{-1}_{\hat{\mathbf{a}}\hat{\mathbf{a}}} (\hat{\mathbf{a}} - \check{\mathbf{a}}_2)$

$$\check{\mathbf{a}} \leftarrow \check{\mathbf{a}}_1$$

Best Integer Equivariant (BIE) [7]

Weighted average

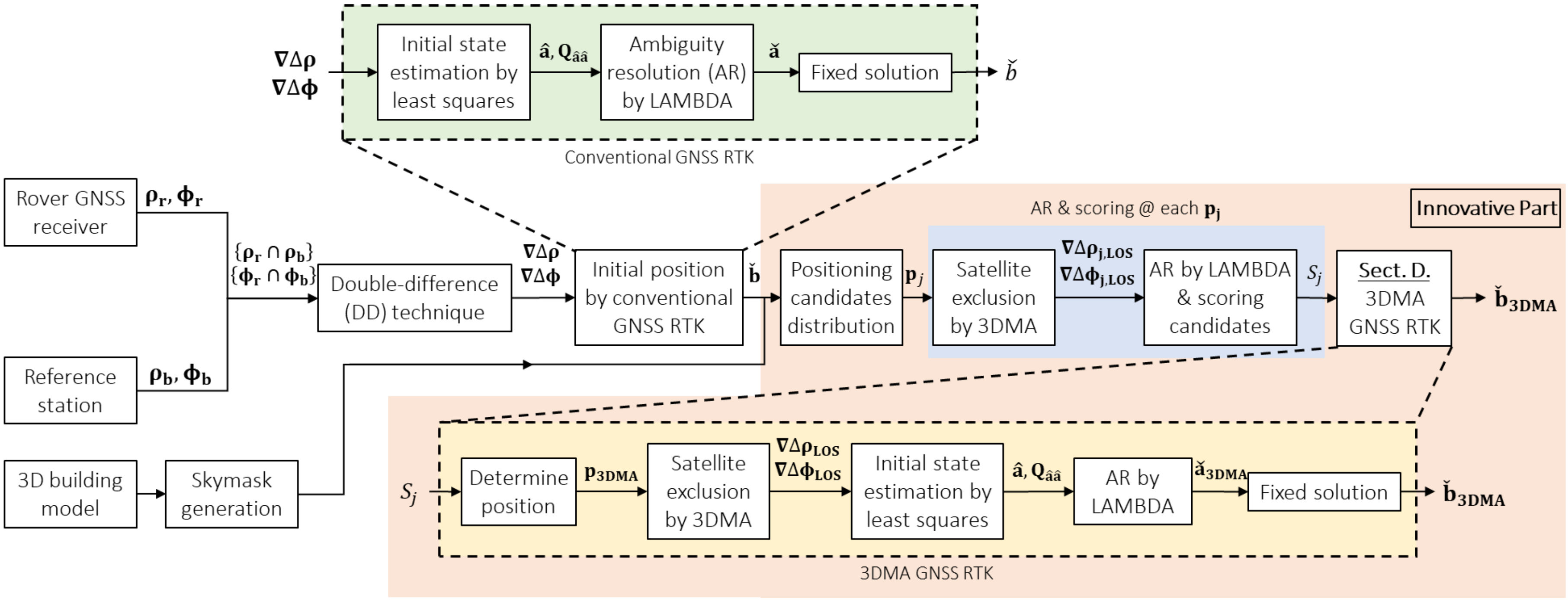
$$\bar{\mathbf{a}} = \sum_{\boldsymbol{z} \in \Theta^{\lambda}_{\hat{a}}} z \frac{\exp\left(-\frac{1}{2}\|\hat{\mathbf{a}} - \boldsymbol{z}\|^2_{\mathbf{Q}_{\hat{\mathbf{a}}\hat{\mathbf{a}}}}\right)}{\sum_{\boldsymbol{z} \in \Theta^{\lambda}_{\hat{a}}} \exp\left(-\frac{1}{2}\|\hat{\mathbf{a}} - \boldsymbol{z}\|^2_{\mathbf{Q}_{\hat{\mathbf{a}}\hat{\mathbf{a}}}}\right)}$$

with $\Theta^{\lambda}_{\hat{a}} = \{\boldsymbol{z} \in Z^n \mid \|\hat{\mathbf{a}} - \boldsymbol{z}\|^2_{\mathbf{Q}_{\hat{\mathbf{a}}\hat{\mathbf{a}}}} < \chi^2\}$

$$\check{\mathbf{a}} \leftarrow \bar{\mathbf{a}}$$

[6] P. J. G. Teunissen, "Least-Squares Estimation of the Integer GPS Ambiguities," *Invited lecture, section IV theory and methodology, IAG general meeting*, 1993.
[7] P. Teunissen, "On the computation of the best integer equivariant estimator," 2005.

3DMA GNSS RTK



3DMA GNSS RTK – Innovative Part

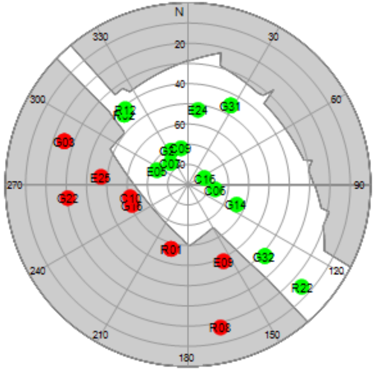
At each candidate, j

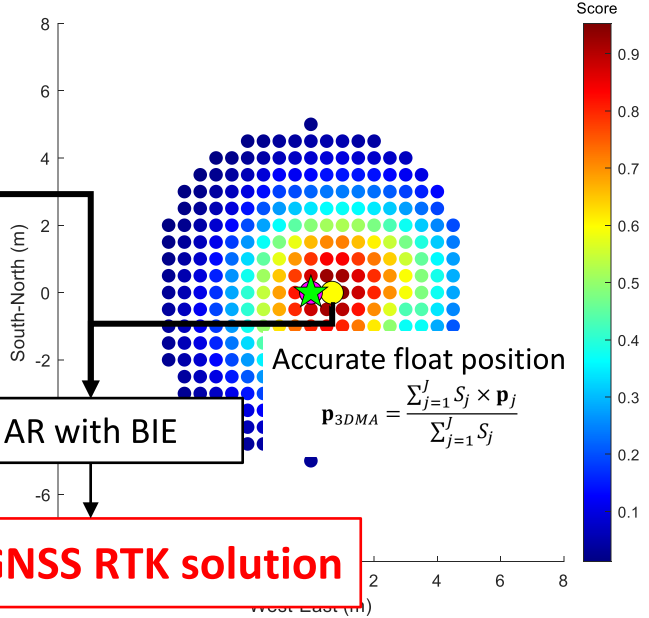
Candidate distribution → NLOS exclusion → AR with BIE → Modelling carrier phase → Scoring candidate

$$\nabla\Delta\phi_j^i = \nabla\Delta D_j^i + \lambda^i \nabla\Delta N_j^i + \varepsilon_{\phi_j^i}$$

$$S_j = \exp\left[-0.5 \times \frac{1}{I}\sum_{i=1}^{I}\left(\nabla\Delta\phi^i - \nabla\Delta\tilde{\phi}_j^i\right)^2\right]$$

5m radius, 50cm separation





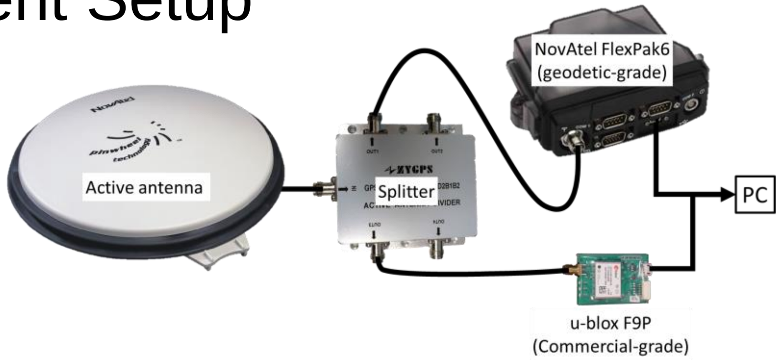
Accurate float position

$$\mathbf{p}_{3DMA} = \frac{\sum_{j=1}^{J} S_j \times \mathbf{p}_j}{\sum_{j=1}^{J} S_j}$$

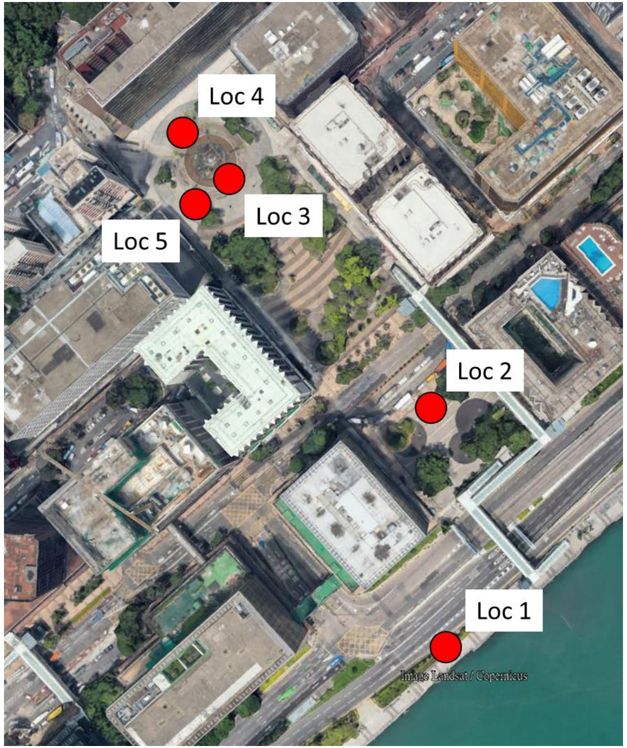
AR with BIE

3DMA GNSS RTK solution

Experiment Setup



Equipment setup



Experiment locations

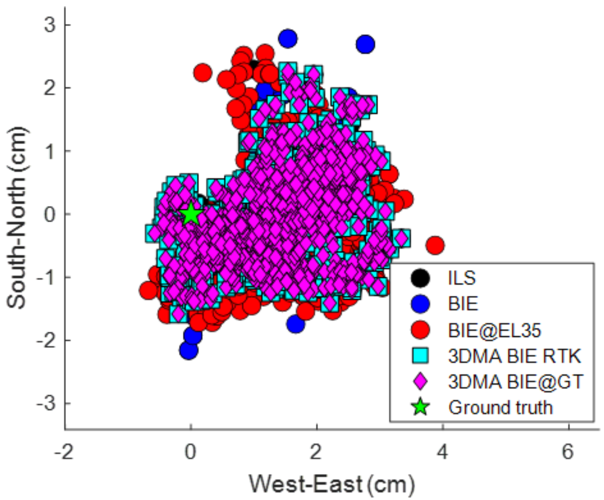
Algorithm	Initial state estimation	AR method	Applying 3DMA	Applying continuous LOS (C-LOS)	Elevation cutoff angle (degree)	C/N_0 cutoff (dBHz)
ILS	Least square	LAMBDA	No	No	15	15
BIE	Least square	BIE	No	No	15	
BIE@EL35	Least square	BIE	No	No	35	
3DMA BIE RTK	Accurate float position	BIE	Yes	Yes	15	
3DMA BIE@GT	Ground truth	BIE	Yes	Yes	15	

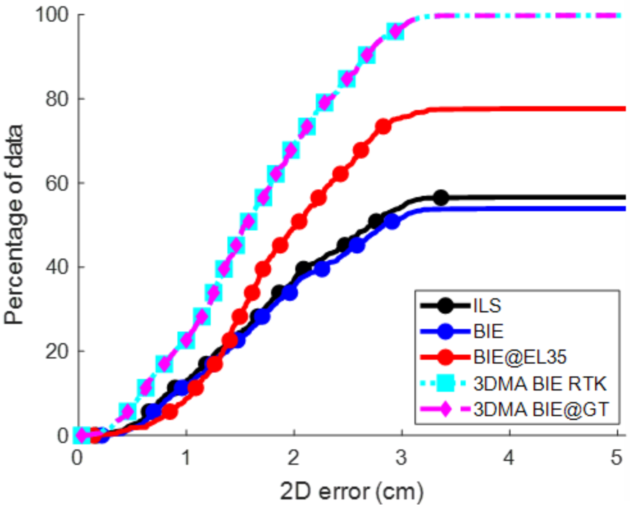
Positioning Results

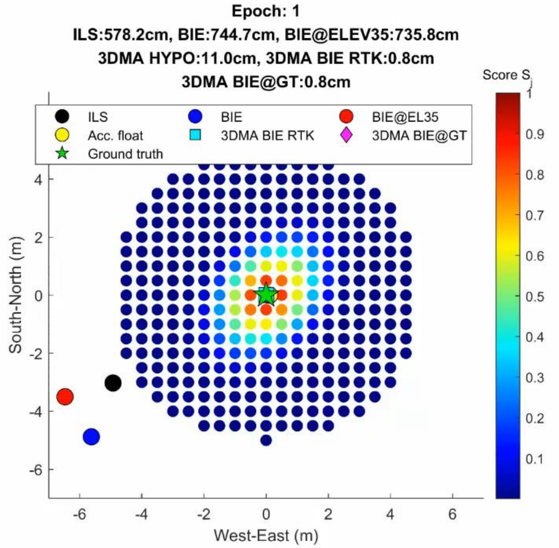
Experiment	Unit: cm	ILS	BIE	BIE@EL35	3DMA BIE RTK	3DMA BIE@GT
1 Relatively opensky	RMS	1.15	1.15	1.50	1.15	1.15
	Mean	1.02	1.02	1.36	1.03	1.03
	STD	0.53	0.53	0.63	0.53	0.53
	Max	2.33	2.33	2.77	2.33	2.33
	Min	0.03	0.03	0.02	0.03	0.03
2 Suburban	RMS	391.36	382.83	306.86	7.47	7.47
	Mean	221.68	214.33	135.43	1.91	1.91
	STD	322.70	317.39	275.51	7.22	7.22
	Max	1254.11	1157.70	885.38	203.68	203.68
	Min	0.16	0.22	0.14	0.03	0.03
3 Urban	RMS	0.90	0.90	0.95	0.93	0.95
	Mean	0.78	0.78	0.86	0.82	0.84
	STD	0.44	0.44	0.41	0.45	0.45
	Max	2.09	2.09	1.97	2.09	2.09
	Min	0.01	0.01	0.02	0.01	0.01
4 Urban, unevenly distributed skymask	RMS	257.25	241.76	30.11	7.95	8.11
	Mean	112.74	126.78	10.31	1.76	2.16
	STD	231.36	205.96	28.30	7.75	7.82
	Max	846.42	593.57	195.78	124.25	124.25
	Min	0.08	0.08	0.06	0.05	0.01
5 Urban, unevenly distributed skymask	RMS	207.98	216.85	62.02	1.93	1.93
	Mean	72.32	74.46	23.43	1.37	1.37
	STD	195.09	203.75	57.45	1.37	1.37
	Max	1228.31	1201.26	295.91	28.00	28.00
	Min	0.03	0.03	0.03	0.03	0.03

Experiment 2

Note: the graph is zoomed in and not all solution are shown







Unit: cm	ILS	BIE	BIE@EL35	3DMA BIE RTK	3DMA BIE@GT
RMS	391.36	382.83	306.86	7.47	7.47
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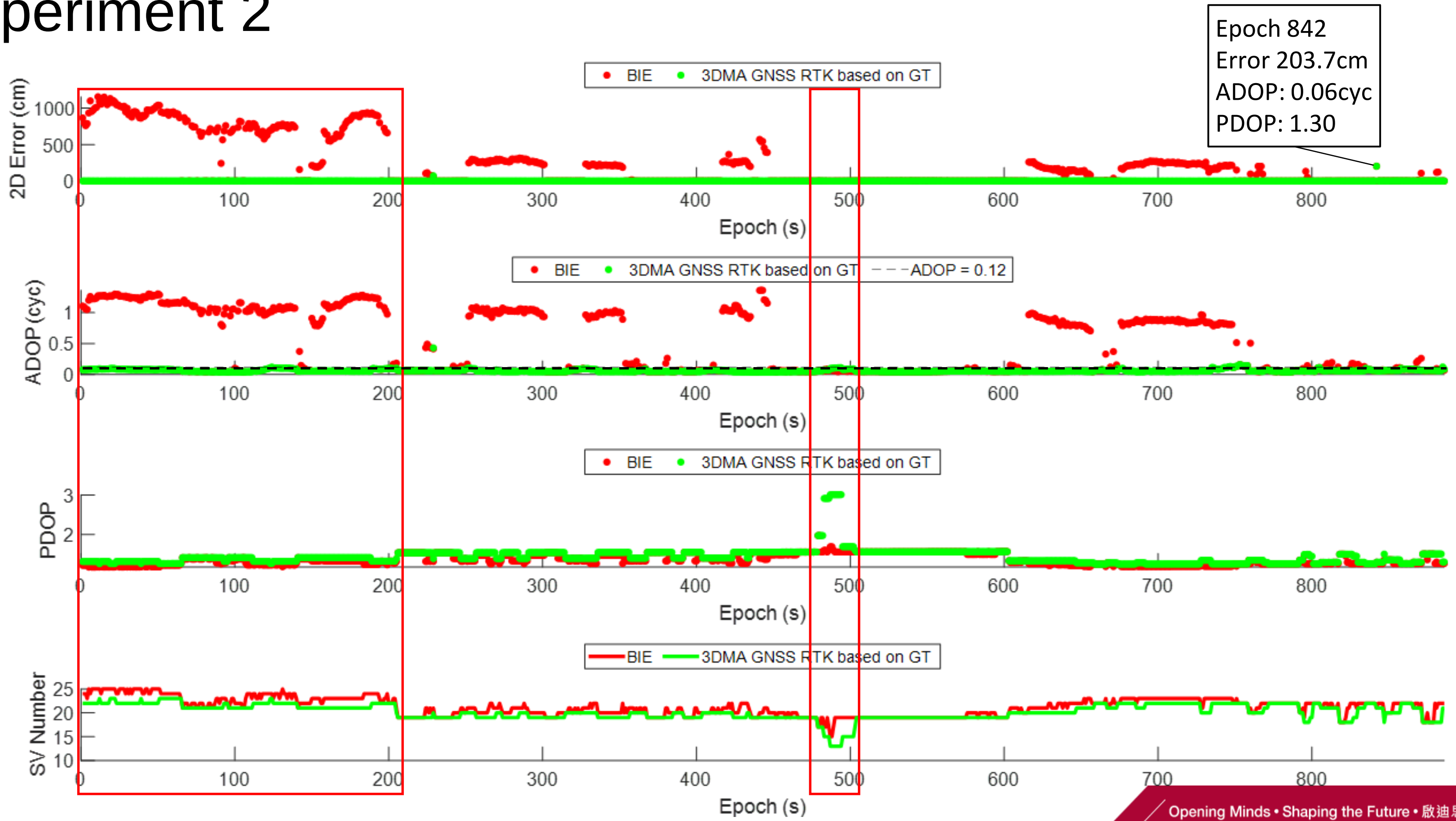
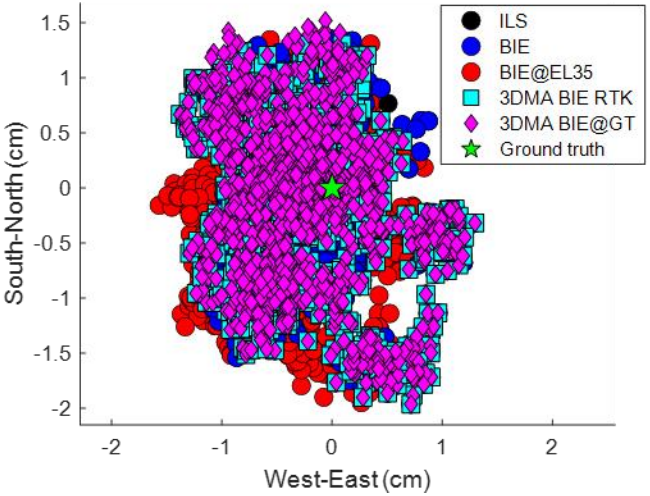
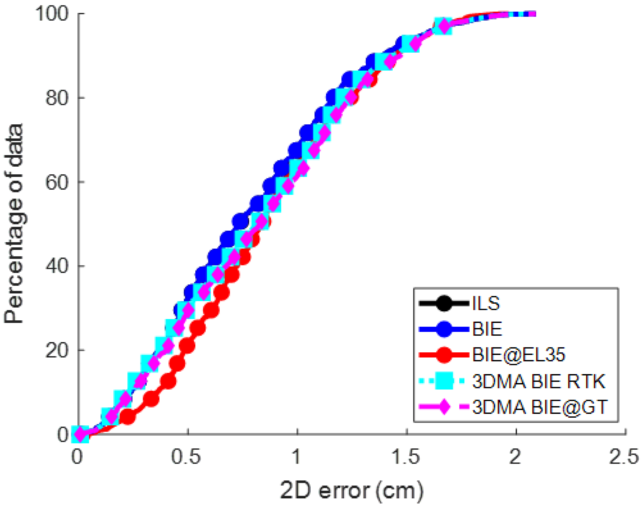
Experiment 2

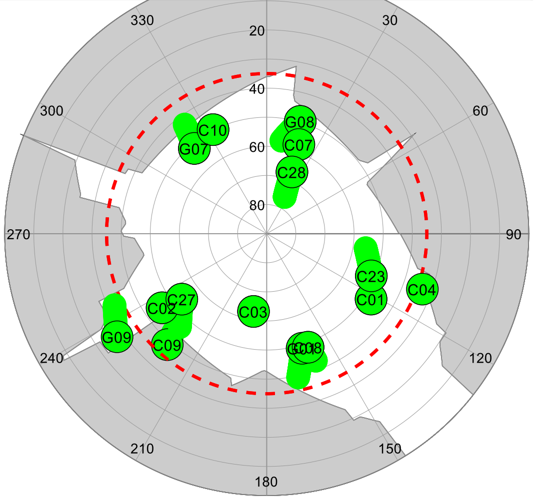


Experiment 3

	Average ADOP	Average PDOP
BIE	0.05	2.01
3DMA GNSS RTK based on GT	0.05	2.03



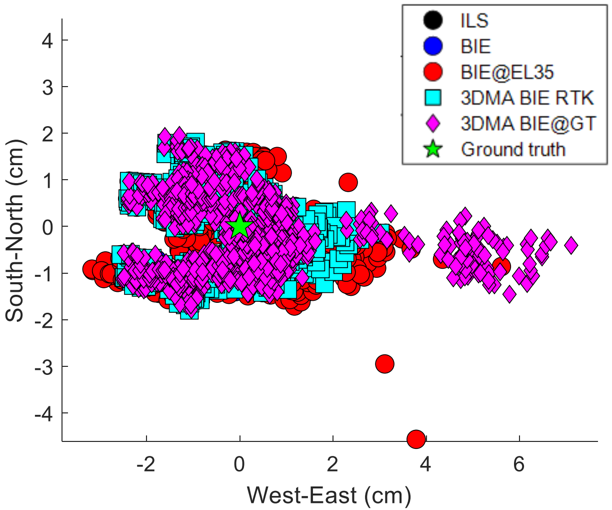


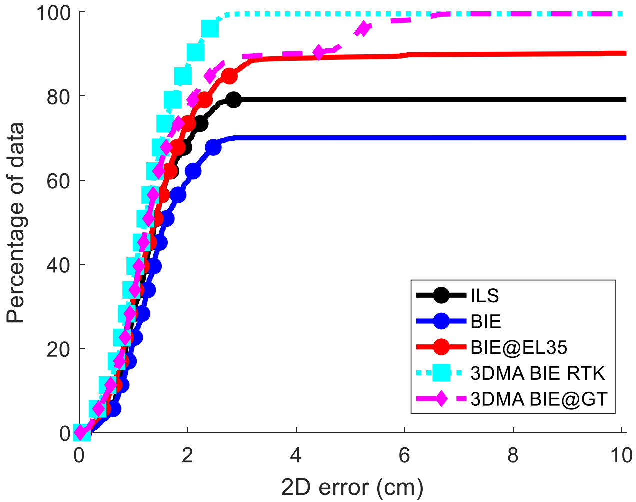


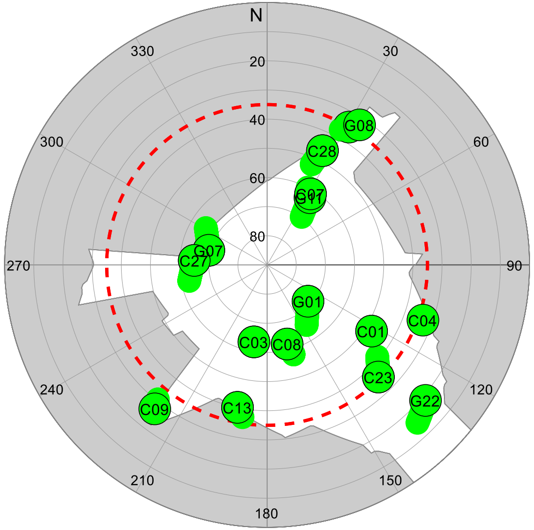
Unit: cm	ILS	BIE	BIE@EL35	3DMA BIE RTK	3DMA BIE@GT
RMS	0.90	0.90	0.95	0.93	0.95
Mean	0.78	0.78	0.86	0.82	0.84
STD	0.44	0.44	0.41	0.45	0.45
Max	2.09	2.09	1.97	2.09	2.09
Min	0.01	0.01	0.02	0.01	0.01

Experiment 4

Note: the graph is zoomed in and not all solution are shown







Unit: cm	ILS	BIE	BIE@EL35	3DMA BIE RTK	3DMA BIE@GT
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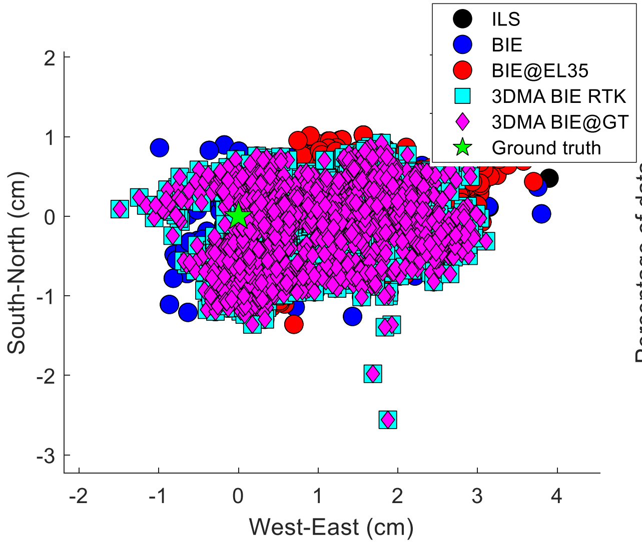
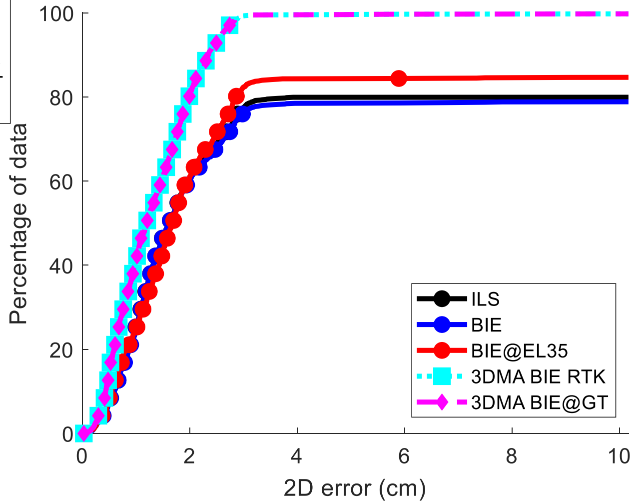
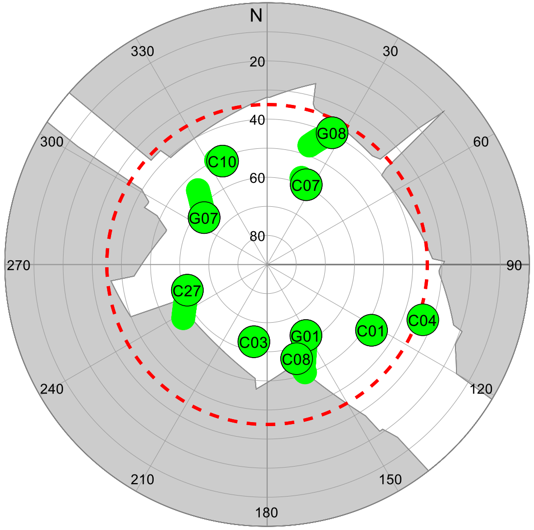
Max	846.42	593.57	195.78	124.25	124.25
Min	0.08	0.08	0.06	0.05	0.01

Experiment 5

Note: the graph is zoomed in and not all solution are shown







Unit: cm	ILS	BIE	BIE@EL35	3DMA BIE RTK	3DMA BIE@GT
RMS	207.98	216.85	62.02	1.93	1.93
Mean	72.32	74.46	23.43	1.37	1.37
STD	195.09	203.75	57.45	1.37	1.37
Max	1228.31	1201.26	295.91	28.00	28.00
Min	0.03	0.03	0.03	0.03	0.03

Conclusions and Future Work

- Healthy satellite is important for ambiguity resolution and GNSS RTK in urban environment
- Exclusion in a dynamic way (by Skymask) is better than that of with a fixed elevation angle threshold
- 10cm accuracy in urban with 3DMA GNSS RTK
- Limitations:
  - Candidates must cover the ground truth
  - Intensive computation load
- Gradient-decent methods is going to adopt

Thank you for your attention

Questions and comments are welcome

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